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(Affiliated to CBSE up to +2 Level)

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Arithmetic Progressions

Question 4. How many terms of AP: 9, 17, 25, … must be taken to give a sum of 636?

Solution:

Given: $a = 9, d = 17 - 9 = 8, \quad S_n = 636$

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow 636 = \frac{n}{2}[2 \times 9 + (n-1)8]$$

$$\Rightarrow 636 \times 2 = n[18 + 8n - 8] \Rightarrow 636 \times 2 = n(10 + 8n)$$

$$\Rightarrow 636 \times 2 = 2n(5 + 4n) \Rightarrow \frac{636 \times 2}{2} = 5n + 4n^2$$

$$\Rightarrow 4n^2 + 5n - 636 = 0 \Rightarrow 4n^2 + 53n - 48n - 636 = 0$$

$$\Rightarrow n(4n + 53) - 12(4n + 53) = 0 \Rightarrow (4n + 53)(n - 12) = 0$$

$$\Rightarrow 4n + 53 = 0 \quad \text{or} \quad n - 12 = 0$$

$$\Rightarrow n = \frac{-53}{4} \text{ (rejected)} \quad \text{or} \quad n = 12$$

Hence, $n = 12$

Question 5.The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution:

Given: $a = 5, l = t_n = 45$ (last term) and $S_n = 400$

$$\therefore \quad \frac{n}{2}[a + l] = 400$$

$$\Rightarrow \frac{n}{2}[5 + 45] = 400$$

$$\Rightarrow \quad n = \frac{400}{25} = 16$$

Now, $t_{16} = 45$

$$\Rightarrow \quad 5 + 15d = 45$$

$$\Rightarrow \quad d = \frac{40}{15} = \frac{8}{3}$$

Thus, $n = \mathbf{16}$ and $d = \mathbf{\frac{8}{3}}$.

Question 6.The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:

Here, $a = 17, a_n = 350, d = 9$

$$a_n = a + (n-1)d \Rightarrow 350 = 17 + (n-1)9$$

$$\Rightarrow 350 - 17 = (n-1)9 \Rightarrow \frac{333}{9} = n - 1 \Rightarrow 37 + 1 = n \Rightarrow n = 38$$

$$S_{38} = \frac{38}{2}[17 + 350] = 19 \times 367 = 6973$$

Question 7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.
Solution:

Given: $d = 7$, $t_{22} = 149$, and $n = 22$

$\therefore \quad t_{22} = a + (22 - 1)d$

$\Rightarrow \quad 149 = a + 21 \times 7$

$\Rightarrow \quad 149 = a + 147 \quad \Rightarrow \quad a = 2$

$\therefore$ The required sum,

$$S_{22} = \frac{22}{2}[2 \times 2 + (22 - 1)7]$$

$$= 11[4 + 147] = 11 \times 151 = \mathbf{1661}.$$

Question 8.Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
Solution:

Given: $a_2 = 14$ and $a_3 = 18$

$\Rightarrow \quad a + d = 14 \quad ...(i) \quad$ and $a + 2d = 18 \quad ...(ii)$

Substracting (*i*) and (*ii*), we get

$$a + 2d - a - d = 18 - 14 \Rightarrow d = 4$$

Since, $\quad a + d = 14 \Rightarrow a + 4 = 14 \Rightarrow a = 14 - 4 \Rightarrow a = 10$

So, $\quad S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1)4]$

$$S_{51} = \frac{51}{2}[20 + 200]$$

$$= \frac{51}{2} \times 220 = 51 \times 110 = 5610$$

Question9.If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.
Solution:

Given: $S_7 = 49$

$\Rightarrow \quad \frac{7}{2}(2a + 6d) = 49$

$\Rightarrow \quad a + 3d = 7 \quad ...(i)$

and $\quad S_{17} = 289$

$\Rightarrow \quad \frac{17}{2}(2a + 16d) = 289$

$\Rightarrow \quad a + 8d = 17 \quad ...(ii)$

Subtracting equation (i) from equation (ii), we get: $d = 2$

Substituting the value of $d = 2$ in equation (i), we get: $a = 7 - 6 = 1$

$\therefore$ Sum of n terms,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n - 1) \times 2] = n^2$$

Hence, the required sum is $\boldsymbol{n^2}$.

Question 10.Show that $a_1, a_2, \ldots\ldots. a_n, \ldots\ldots$ form an AP where an is defined as below:

(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Solution:

(i) $a_n = 3 + 4n$

Putting $n = 1, 2, 3, \ldots\ldots$

$$a_1 = 3 + 4 \times 1 = 3 + 4 = 7$$
$$a_2 = 3 + 4 \times 2 = 3 + 8 = 11$$
$$d = a_2 - a_1 = 11 - 7 = 4$$
$$a_3 = 3 + 4 \times 3 = 3 + 12 = 15$$
$$d = a_3 - a_2 = 15 - 11 = 4$$

Thus, the sequence 7, 11, 15, ... is an AP.

$$S_{15} = \frac{15}{2}[2 \times 7 + (15 - 1) \times 4] = \frac{15}{2}[14 + 56] = \frac{15}{2} \times 70 = 525$$

(ii) $a_n = 9 - 5n$

Putting $n = 1, 2, 3, \ldots$

$$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$$
$$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$$
$$d = a_2 - a_1 = -1 - 4 = -5$$
$$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$$
$$d = a_3 - a_2 = -6 - (-1) = -6 + 1 = -5$$

Thus, the sequence 4, –1, –6, ..., is an A.P.

$$a = 4, d = -1 - 4 = -5$$
$$S_{15} = \frac{15}{2}[2 \times 4 + (15 - 1)(-5)] = \frac{15}{2}[8 - 70]$$
$$= \frac{15}{2} \times (-62) = -465$$

Question 11.If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Solution:

We have $S_n = 4n - n^2$ ...(i)

Putting $n = 1$ in equation (i), we get:

$S_1 = 4 - 1 = \mathbf{3}$.

In an AP, $S_1 = t_1 = a$. So $a = 3$.

Putting $n = 2$ in equation (i), we get:

$S_2 = 4 \times 2 - (2)^2 = \mathbf{4}$.

$t_2 = S_2 - S_1 = 4 - 3 = \mathbf{1}$

Putting $n = 3$ in equation (i), we get:

$S_3 = 4 \times 3 - 3^2 = 12 - 9$

$t_3 = S_3 - S_2 = 3 - 4 = -1$

$\therefore$ Third term is $\mathbf{-1}$.

Putting $n = 9$ in equation (i), we get:

$S_9 = 4 \times 9 - 9^2 = 36 - 81 = -45$

Similarly, $S_{10} = 4 \times 10 - 10^2 = 40 - 100 = -60$

$t_{10} = S_{10} - S_9 = -60 - (-45) = -60 + 45 = \mathbf{-15}$

$\therefore$ Tenth term is –15.

Here, common difference, $d = t_2 - t_1$

$= 1 - 3 = -2$

$\therefore$ nth term, $t_n = a + (n - 1)d$

$= 3 + (n - 1)(-2)$

$= 3 - 2n + 2 = \mathbf{5 - 2n}$.